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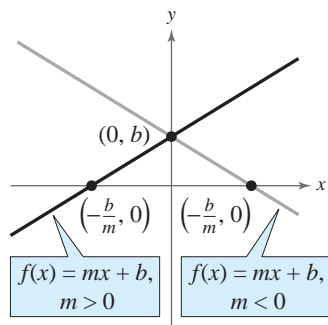
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## GRAPHS OF PARENT FUNCTIONS

### Linear Function

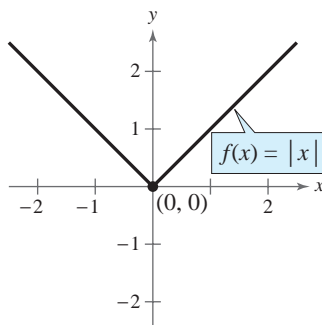
$$f(x) = mx + b$$



Domain:  $(-\infty, \infty)$   
 Range:  $(-\infty, \infty)$   
 x-intercept:  $(-b/m, 0)$   
 y-intercept:  $(0, b)$   
 Increasing when  $m > 0$   
 Decreasing when  $m < 0$

### Absolute Value Function

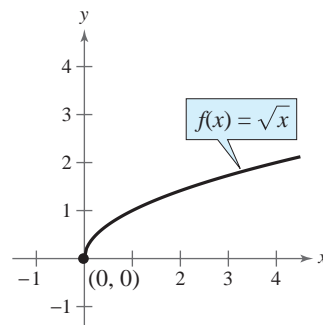
$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Domain:  $(-\infty, \infty)$   
 Range:  $[0, \infty)$   
 Intercept:  $(0, 0)$   
 Decreasing on  $(-\infty, 0)$   
 Increasing on  $(0, \infty)$   
 Even function  
 y-axis symmetry

### Square Root Function

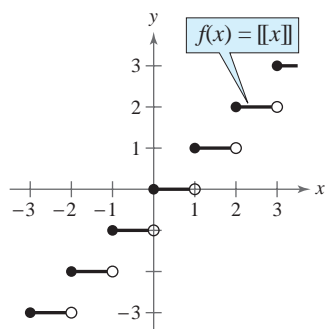
$$f(x) = \sqrt{x}$$



Domain:  $[0, \infty)$   
 Range:  $[0, \infty)$   
 Intercept:  $(0, 0)$   
 Increasing on  $(0, \infty)$

### Greatest Integer Function

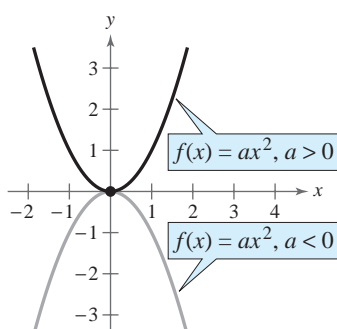
$$f(x) = \llbracket x \rrbracket$$



Domain:  $(-\infty, \infty)$   
 Range: the set of integers  
 x-intercepts: in the interval  $[0, 1)$   
 y-intercept:  $(0, 0)$   
 Constant between each pair of consecutive integers  
 Jumps vertically one unit at each integer value

### Quadratic (Squaring) Function

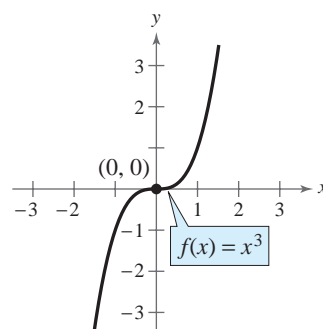
$$f(x) = ax^2$$



Domain:  $(-\infty, \infty)$   
 Range ( $a > 0$ ):  $[0, \infty)$   
 Range ( $a < 0$ ):  $(-\infty, 0]$   
 Intercept:  $(0, 0)$   
 Decreasing on  $(-\infty, 0)$  for  $a > 0$   
 Increasing on  $(0, \infty)$  for  $a > 0$   
 Increasing on  $(-\infty, 0)$  for  $a < 0$   
 Decreasing on  $(0, \infty)$  for  $a < 0$   
 Even function  
 y-axis symmetry  
 Relative minimum ( $a > 0$ ),  
 relative maximum ( $a < 0$ ),  
 or vertex:  $(0, 0)$

### Cubic Function

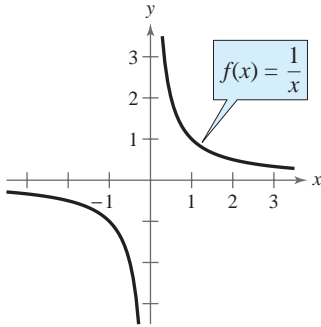
$$f(x) = x^3$$



Domain:  $(-\infty, \infty)$   
 Range:  $(-\infty, \infty)$   
 Intercept:  $(0, 0)$   
 Increasing on  $(-\infty, \infty)$   
 Odd function  
 Origin symmetry

### Rational (Reciprocal) Function

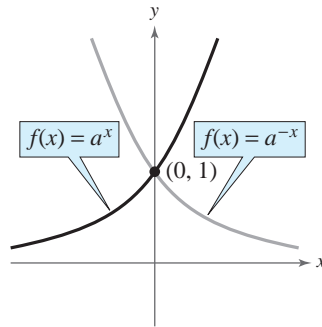
$$f(x) = \frac{1}{x}$$



Domain:  $(-\infty, 0) \cup (0, \infty)$   
 Range:  $(-\infty, 0) \cup (0, \infty)$   
 No intercepts  
 Decreasing on  $(-\infty, 0)$  and  $(0, \infty)$   
 Odd function  
 Origin symmetry  
 Vertical asymptote:  $y$ -axis  
 Horizontal asymptote:  $x$ -axis

### Exponential Function

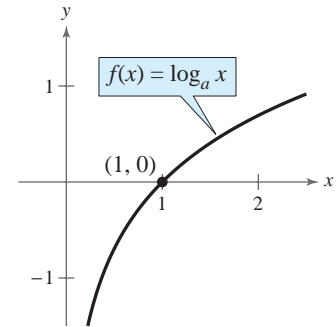
$$f(x) = a^x, a > 0, a \neq 1$$



Domain:  $(-\infty, \infty)$   
 Range:  $(0, \infty)$   
 Intercept:  $(0, 1)$   
 Increasing on  $(-\infty, \infty)$   
 for  $f(x) = a^x$   
 Decreasing on  $(-\infty, \infty)$   
 for  $f(x) = a^{-x}$   
 Horizontal asymptote:  $x$ -axis  
 Continuous

### Logarithmic Function

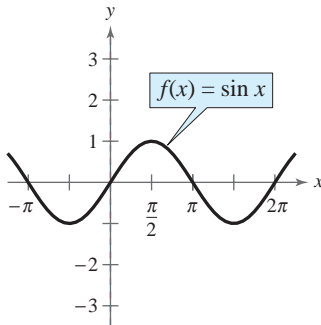
$$f(x) = \log_a x, a > 0, a \neq 1$$



Domain:  $(0, \infty)$   
 Range:  $(-\infty, \infty)$   
 Intercept:  $(1, 0)$   
 Increasing on  $(0, \infty)$   
 Vertical asymptote:  $y$ -axis  
 Continuous  
 Reflection of graph of  $f(x) = a^x$   
 in the line  $y = x$

### Sine Function

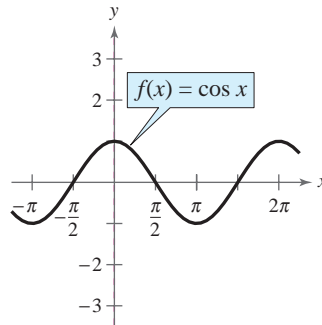
$$f(x) = \sin x$$



Domain:  $(-\infty, \infty)$   
 Range:  $[-1, 1]$   
 Period:  $2\pi$   
 $x$ -intercepts:  $(n\pi, 0)$   
 $y$ -intercept:  $(0, 0)$   
 Odd function  
 Origin symmetry

### Cosine Function

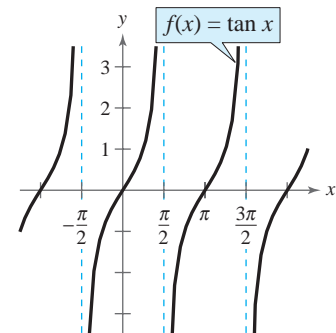
$$f(x) = \cos x$$



Domain:  $(-\infty, \infty)$   
 Range:  $[-1, 1]$   
 Period:  $2\pi$   
 $x$ -intercepts:  $(\frac{\pi}{2} + n\pi, 0)$   
 $y$ -intercept:  $(0, 1)$   
 Even function  
 $y$ -axis symmetry

### Tangent Function

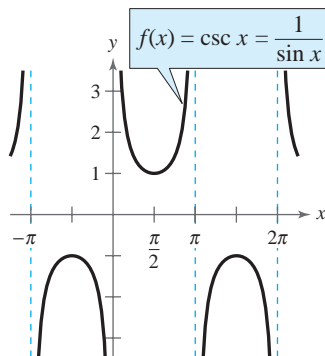
$$f(x) = \tan x$$



Domain: all  $x \neq \frac{\pi}{2} + n\pi$   
 Range:  $(-\infty, \infty)$   
 Period:  $\pi$   
 $x$ -intercepts:  $(n\pi, 0)$   
 $y$ -intercept:  $(0, 0)$   
 Vertical asymptotes:  
 $x = \frac{\pi}{2} + n\pi$   
 Odd function  
 Origin symmetry

### Cosecant Function

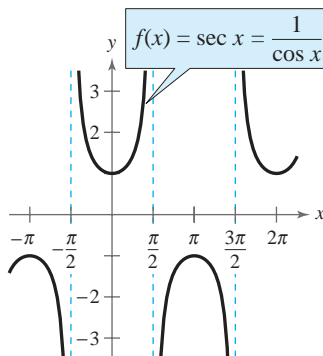
$$f(x) = \csc x$$



Domain: all  $x \neq n\pi$   
 Range:  $(-\infty, -1] \cup [1, \infty)$   
 Period:  $2\pi$   
 No intercepts  
 Vertical asymptotes:  $x = n\pi$   
 Odd function  
 Origin symmetry

### Secant Function

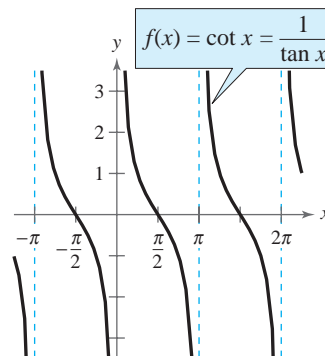
$$f(x) = \sec x$$



Domain: all  $x \neq \frac{\pi}{2} + n\pi$   
 Range:  $(-\infty, -1] \cup [1, \infty)$   
 Period:  $2\pi$   
 y-intercept:  $(0, 1)$   
 Vertical asymptotes:  
 $x = \frac{\pi}{2} + n\pi$   
 Even function  
 y-axis symmetry

### Cotangent Function

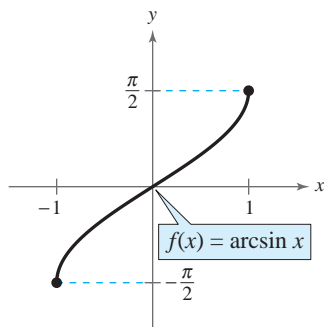
$$f(x) = \cot x$$



Domain: all  $x \neq n\pi$   
 Range:  $(-\infty, \infty)$   
 Period:  $\pi$   
 x-intercepts:  $(\frac{\pi}{2} + n\pi, 0)$   
 Vertical asymptotes:  $x = n\pi$   
 Odd function  
 Origin symmetry

### Inverse Sine Function

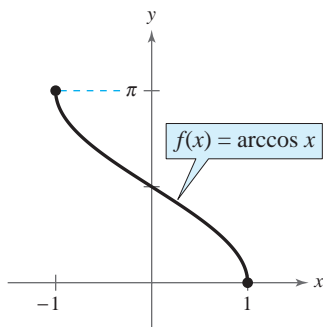
$$f(x) = \arcsin x$$



Domain:  $[-1, 1]$   
 Range:  $[-\frac{\pi}{2}, \frac{\pi}{2}]$   
 Intercept:  $(0, 0)$   
 Odd function  
 Origin symmetry

### Inverse Cosine Function

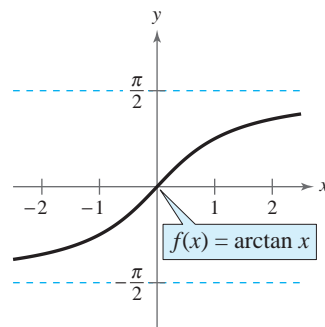
$$f(x) = \arccos x$$



Domain:  $[-1, 1]$   
 Range:  $[0, \pi]$   
 y-intercept:  $(0, \frac{\pi}{2})$

### Inverse Tangent Function

$$f(x) = \arctan x$$



Domain:  $(-\infty, \infty)$   
 Range:  $(-\frac{\pi}{2}, \frac{\pi}{2})$   
 Intercept:  $(0, 0)$   
 Horizontal asymptotes:  
 $y = \pm \frac{\pi}{2}$   
 Odd function  
 Origin symmetry